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Divergence of Compound Stings

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DIVERGENCE OF COMPOUND STINGS

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INTRODUCTION

This report documents an analytical technique that was originally developed to evaluate the critical sting divergence dynamic pressure of an existing wind tunnel model support system that included a dummy sting strut that was added to study sting interference effects. The critical sting divergence pressure can be defined as the dynamic pressure where the aerodynamic loads on the model(s) exceed the elastic restoring force developed by the model support system. The analytical technique that is included in this report can also be applied to evaluate the critical sting divergence pressure of other types of compound sting systems. As used in this report, a compound sting system is categorized as either a parallel or a tandem model support system, see Figures 1 and 2, and the implied reference is relative to the respective arrangement of the models. The same analytical technique applies to both arrangements and the only difference in evaluating results for the different sting systems would be from the numerical differences in the deflection influence coefficients and/or aerodynamic loads that are used for input.

In general, a compound sting systems consist of two model support systems that are coupled together such that the loads on one of the models can affect the deflections of the other. If considered alone, the aerodynamic loads and elastic forces on either model and model support systems are not as critical as when they are combined to form a compound sting system. Previously used analytical techniques do not account for simultaneously considering the aeroelastic coupling between adjacent lifting bodies. The worst case condition is when the elasticity of the stings are coupled together to form a "compound sting system" and the result is a lowering in the sting divergence dynamic pressure which is the subject of this report.

To evaluate the minimum sting divergence dynamic pressure of a compound sting system, using the equations in this report, requires that the flexibility of the model support system and that the aerodynamic loads be

defined in advance. The specific input that is required include the rotational deflection influence coefficients that define the model support system and the aerodynamic characteristics that produce the loads on the models. The rotational deflection influence coefficients must be determined in terms of the normal force and pitch moment and must be referenced to the same location about which the aerodynamic loads are defined. In most cases the aerodynamic loads that act in the vertical plane are sufficient but if the loads in the horizontal plane are to be included, they should be combined with the vertical loads and a worst case load condition determined. Typically the loads that are used in the equations in this report are the rate of change of normal force per unit angle of attack and the rate of change of the pitch moment per unit angle of attack which all act in the vertical plane. Generally the coefficients that define the rotational flexibility of the model support system are determined for the extreme forward end where the model(s) attach to the model support system and for consistency the aerodynamic loads must also be defined at this location. If the aerodynamic loads are referenced about some other location, the loads must be transposed from the reference center of the model(s) to the extreme forward end of the model support system.

A sample problem is included in Appendix B to illustrate how the results in this report can be applied to determine the critical sting divergence dynamic pressure of a typical compound sting system and includes the above mentioned process of transposing the aerodynamic loads.

DERIVATION OF GOVERNING EQUATION

The deflections of two bodies that are supported by a sting system, such as shown in Figure 3, can be expressed in the form of a matrix equation. The loads are due to the aerodynamics that act on the bodies and in general are dependent on the angles of attack of the bodies. Such matrix equations where the loads are deflection dependent are known as characteristic value problems and can be solved for the characteristic value that satisfies the matrix equation. For this application the characteristic value would be the dynamic pressure that results in divergence of the sting system. The following is the general matrix equation, that defines the aeroelastic deflections of two bodies that each have a translational and a rotational degree of freedom, in terms of a deflection influence coefficient matrix and an aerodynamic load vector:

$$\begin{bmatrix} y_1 \\ \theta_1 \\ y_2 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix} \times \begin{bmatrix} N_1 \\ M_1 \\ N_2 \\ M_2 \end{bmatrix}$$

The coefficients in the aerodynamic load vector on the right side of the above matrix equation can be defined in terms of the rate of change in the lift and pitch coefficients, respective reference areas and chords, angle of attack of the respective bodies, and a general dynamic pressure term. This description of the aerodynamic load vector can be expressed as the product of an aerodynamic load matrix and the deflection vector as follows:

$$\begin{bmatrix} N_1 \\ M_1 \\ N_2 \\ M_2 \end{bmatrix} = \begin{bmatrix} 0 & (C_{N\alpha}S)_1Q & 0 & 0 \\ 0 & (C_{M\alpha}Sh)_1Q & 0 & 0 \\ 0 & 0 & (C_{N\alpha}S)_2Q & 0 \\ 0 & 0 & (C_{M\alpha}Sh)_2Q & 0 \end{bmatrix} \times \begin{bmatrix} y_1 \\ \theta_1 \\ y_2 \\ \theta_2 \end{bmatrix}$$

Substituting this description of the aerodynamic load vector into the general matrix equation that defines the aeroelastic deflections provides

$$\begin{bmatrix} y_1 \\ \theta_1 \\ y_2 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix} \times \begin{bmatrix} 0 & (C_{N\alpha}S)_1Q & 0 & 0 \\ 0 & (C_{M\alpha}S)_1Q & 0 & 0 \\ 0 & 0 & (C_{N\alpha}S)_2Q & 0 \\ 0 & 0 & (C_{M\alpha}S)_2Q & 0 \end{bmatrix} \times \begin{bmatrix} y_1 \\ \theta_1 \\ y_2 \\ \theta_2 \end{bmatrix}$$

Combining the square matrices produces the following characteristic matrix equation that can be solved for the characteristic value Q

$$\begin{bmatrix} y_1 \\ \theta_1 \\ y_2 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} 0 & a_{11}(C_{N\alpha}S)_1Q + a_{12}(C_{M\alpha}Sh)_1Q & 0 & a_{13}(C_{N\alpha}S)_2Q + a_{14}(C_{M\alpha}Sh)_2Q \\ 0 & a_{21}(C_{N\alpha}S)_1Q + a_{22}(C_{M\alpha}Sh)_1Q & 0 & a_{23}(C_{N\alpha}S)_2Q + a_{24}(C_{M\alpha}Sh)_2Q \\ 0 & a_{31}(C_{N\alpha}S)_1Q + a_{32}(C_{M\alpha}Sh)_1Q & 0 & a_{33}(C_{N\alpha}S)_2Q + a_{34}(C_{M\alpha}Sh)_2Q \\ 0 & a_{41}(C_{N\alpha}S)_1Q + a_{42}(C_{M\alpha}Sh)_1Q & 0 & a_{43}(C_{N\alpha}S)_2Q + a_{44}(C_{M\alpha}Sh)_2Q \end{bmatrix} \times \begin{bmatrix} y_1 \\ \theta_1 \\ y_2 \\ \theta_2 \end{bmatrix}$$

The following alternate form of the characteristic matrix equation is obtained by transferring the left displacement vector to the right hand side of the equation, premultiplying it by an identity matrix, and then additively combining with the existing square matrix to yield

$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 & a_{11}(C_{N\alpha}S)_1Q + a_{12}(C_{M\alpha}Sh)_1Q & 0 & a_{13}(C_{N\alpha}S)_2Q + a_{14}(C_{M\alpha}Sh)_2Q \\ 0 & a_{21}(C_{N\alpha}S)_1Q + a_{22}(C_{M\alpha}Sh)_1Q - 1 & 0 & a_{23}(C_{N\alpha}S)_2Q + a_{24}(C_{M\alpha}Sh)_2Q \\ 0 & a_{31}(C_{N\alpha}S)_1Q + a_{32}(C_{M\alpha}Sh)_1Q & -1 & a_{33}(C_{N\alpha}S)_2Q + a_{34}(C_{M\alpha}Sh)_2Q \\ 0 & a_{41}(C_{N\alpha}S)_1Q + a_{42}(C_{M\alpha}Sh)_1Q & 0 & a_{43}(C_{N\alpha}S)_2Q + a_{44}(C_{M\alpha}Sh)_2Q - 1 \end{bmatrix} \times \begin{bmatrix} y_1 \\ \theta_1 \\ y_2 \\ \theta_2 \end{bmatrix}$$

For a nontrivial solution, the determinant of the square coefficient matrix must vanish. A physical interpretation of this is that the characteristic value(s) are to be determined for which the aerodynamic loads on the models are identical to the elastic restoring forces of the model support system.

For a given model support system, the deflection influence coefficients are constant and for the models the maximum slope for the normal force and pitch moment coefficient curves are selected for use with the reference planform area and reference chord which are also fixed parameters. This leaves the dynamic pressure, Q , as the independent variable for making the determinant vanish and it is identified as the divergence dynamic pressure, Q_{div} . To solve for Q_{div} the determinant is set equal to zero, expanded into a scalar equation, and solved for the value of Q that makes the equation identically zero. Setting the determinant of the coefficient matrix equal to zero,

$$\begin{vmatrix} -1 & a_{11}(C_{N\alpha}S)_1Q + a_{12}(C_{M\alpha}Sh)_1Q & 0 & a_{13}(C_{N\alpha}S)_2Q + a_{14}(C_{M\alpha}Sh)_2Q \\ 0 & a_{21}(C_{N\alpha}S)_1Q + a_{22}(C_{M\alpha}Sh)_1Q - 1 & 0 & a_{23}(C_{N\alpha}S)_2Q + a_{24}(C_{M\alpha}Sh)_2Q \\ 0 & a_{31}(C_{N\alpha}S)_1Q + a_{32}(C_{M\alpha}Sh)_1Q & -1 & a_{33}(C_{N\alpha}S)_2Q + a_{34}(C_{M\alpha}Sh)_2Q \\ 0 & a_{41}(C_{N\alpha}S)_1Q + a_{42}(C_{M\alpha}Sh)_1Q & 0 & a_{43}(C_{N\alpha}S)_2Q + a_{44}(C_{M\alpha}Sh)_2Q - 1 \end{vmatrix} = 0$$

For the first expansion of the determinant, the elements of the first column of the array are postmultiplied by the respective cofactors to obtain the following:

$$\begin{aligned} & (-1) \begin{vmatrix} a_{21}(C_{N\alpha}S)_1Q + a_{22}(C_{M\alpha}Sh)_1Q - 1 & 0 & a_{23}(C_{N\alpha}S)_2Q + a_{24}(C_{M\alpha}Sh)_2Q \\ a_{31}(C_{N\alpha}S)_1Q + a_{32}(C_{M\alpha}Sh)_1Q & -1 & a_{33}(C_{N\alpha}S)_2Q + a_{34}(C_{M\alpha}Sh)_2Q \\ a_{41}(C_{N\alpha}S)_1Q + a_{42}(C_{M\alpha}Sh)_1Q & 0 & a_{43}(C_{N\alpha}S)_2Q + a_{44}(C_{M\alpha}Sh)_2Q - 1 \end{vmatrix} + \\ & - (0) \begin{vmatrix} a_{11}(C_{N\alpha}S)_1Q + a_{12}(C_{M\alpha}Sh)_1Q & 0 & a_{13}(C_{N\alpha}S)_2Q + a_{14}(C_{M\alpha}Sh)_2Q \\ a_{31}(C_{N\alpha}S)_1Q + a_{32}(C_{M\alpha}Sh)_1Q & -1 & a_{33}(C_{N\alpha}S)_2Q + a_{34}(C_{M\alpha}Sh)_2Q \\ a_{41}(C_{N\alpha}S)_1Q + a_{42}(C_{M\alpha}Sh)_1Q & 0 & a_{43}(C_{N\alpha}S)_2Q + a_{44}(C_{M\alpha}Sh)_2Q - 1 \end{vmatrix} + \\ & + (0) \begin{vmatrix} a_{11}(C_{N\alpha}S)_1Q + a_{12}(C_{M\alpha}Sh)_1Q & 0 & a_{13}(C_{N\alpha}S)_2Q + a_{14}(C_{M\alpha}Sh)_2Q \\ a_{21}(C_{N\alpha}S)_1Q + a_{22}(C_{M\alpha}Sh)_1Q - 1 & 0 & a_{23}(C_{N\alpha}S)_2Q + a_{24}(C_{M\alpha}Sh)_2Q \\ a_{41}(C_{N\alpha}S)_1Q + a_{42}(C_{M\alpha}Sh)_1Q & 0 & a_{43}(C_{N\alpha}S)_2Q + a_{44}(C_{M\alpha}Sh)_2Q - 1 \end{vmatrix} + \\ & - (0) \begin{vmatrix} a_{11}(C_{N\alpha}S)_1Q + a_{12}(C_{M\alpha}Sh)_1Q & 0 & a_{13}(C_{N\alpha}S)_2Q + a_{14}(C_{M\alpha}Sh)_2Q \\ a_{21}(C_{N\alpha}S)_1Q + a_{22}(C_{M\alpha}Sh)_1Q - 1 & 0 & a_{23}(C_{N\alpha}S)_2Q + a_{24}(C_{M\alpha}Sh)_2Q \\ a_{31}(C_{N\alpha}S)_1Q + a_{32}(C_{M\alpha}Sh)_1Q & -1 & a_{33}(C_{N\alpha}S)_2Q + a_{34}(C_{M\alpha}Sh)_2Q \end{vmatrix} = 0 \end{aligned}$$

Since the coefficients that premultiply the later three determinants listed above are zero, the product of these coefficients with any resulting subsequent expansions of the determinants would also be zero. This leaves the product of the first coefficient and determinant as the only nonzero product in the above expression. Expanding this first determinant in terms the products of the elements in the upper row postmultiplied by the respective cofactors provides the following:

$$\begin{aligned}
& (-1) \left\{ \left[a_{21}(C_{N\alpha}S)_1Q + a_{22}(C_{M\alpha}Sh)_1Q - 1 \right] \begin{vmatrix} -1 & a_{33}(C_{N\alpha}S)_2Q + a_{34}(C_{M\alpha}Sh)_2Q \\ 0 & a_{43}(C_{N\alpha}S)_2Q + a_{44}(C_{M\alpha}Sh)_2Q - 1 \end{vmatrix} + \right. \\
& \quad - (0) \begin{vmatrix} a_{31}(C_{N\alpha}S)_1Q + a_{32}(C_{M\alpha}Sh)_1Q & a_{33}(C_{N\alpha}S)_2Q + a_{34}(C_{M\alpha}Sh)_2Q \\ a_{41}(C_{N\alpha}S)_1Q + a_{42}(C_{M\alpha}Sh)_1Q & a_{43}(C_{N\alpha}S)_2Q + a_{44}(C_{M\alpha}Sh)_2Q - 1 \end{vmatrix} + \\
& \quad \left. + [a_{23}(C_{N\alpha}S)_2Q + a_{24}(C_{M\alpha}Sh)_2Q] \begin{vmatrix} a_{31}(C_{N\alpha}S)_1Q + a_{32}(C_{M\alpha}Sh)_1Q & -1 \\ a_{41}(C_{N\alpha}S)_1Q + a_{42}(C_{M\alpha}Sh)_1Q & 0 \end{vmatrix} \right\} = 0
\end{aligned}$$

Since the coefficient that premultiplies the second determinant is zero, its product would be zero and the expansion of the above expression would only include terms from the expansion of the first and third determinant. The following scalar equation results from expanding the first and third determinant and only including the nonzero terms:

$$\begin{aligned}
& (-1) \left\{ [a_{21}(C_{N\alpha}S)_1Q + a_{22}(C_{M\alpha}Sh)_1Q - 1] (-1) [a_{43}(C_{N\alpha}S)_2Q + a_{44}(C_{M\alpha}Sh)_2Q - 1] \right. \\
& \quad \left. - [a_{23}(C_{N\alpha}S)_2Q + a_{24}(C_{M\alpha}Sh)_2Q] [a_{41}(C_{N\alpha}S)_1Q + a_{42}(C_{M\alpha}Sh)_1Q] (-1) \right\} = 0
\end{aligned}$$

Simplifying

$$\begin{aligned}
& [a_{21}(C_{N\alpha}S)_1Q + a_{22}(C_{M\alpha}Sh)_1Q - 1] [a_{43}(C_{N\alpha}S)_2Q + a_{44}(C_{M\alpha}Sh)_2Q - 1] \\
& \quad - [a_{23}(C_{N\alpha}S)_2Q + a_{24}(C_{M\alpha}Sh)_2Q] [a_{41}(C_{N\alpha}S)_1Q + a_{42}(C_{M\alpha}Sh)_1Q] = 0
\end{aligned}$$

Carrying out the multiplications and regrouping into terms of equal powers of the dynamic pressure, Q , provides the following quadratic equation:

$$\left\{ \begin{aligned} & a_{21}a_{43}(C_{N\alpha}S)_1(C_{N\alpha}S)_2 + a_{21}a_{44}(C_{N\alpha}S)_1(C_{M\alpha}Sh)_2 + a_{22}a_{43}(C_{M\alpha}Sh)_1(C_{N\alpha}S)_2 \\ & + a_{22}a_{44}(C_{M\alpha}Sh)_1(C_{M\alpha}Sh)_2 - a_{23}a_{41}(C_{N\alpha}S)_1(C_{N\alpha}S)_2 - a_{23}a_{42}(C_{M\alpha}Sh)_1(C_{N\alpha}S)_2 \\ & - a_{24}a_{41}(C_{N\alpha}S)_1(C_{M\alpha}Sh)_2 - a_{24}a_{42}(C_{M\alpha}Sh)_1(C_{M\alpha}Sh)_2 \end{aligned} \right\} Q^2 + \\ - \left\{ a_{21}(C_{N\alpha}S)_1 + a_{22}(C_{M\alpha}Sh)_1 + a_{43}(C_{N\alpha}S)_2 + a_{44}(C_{M\alpha}Sh)_2 \right\} Q + 1 = 0$$

The roots to the quadratic equation can be determined from the quadratic formula and these roots, or dynamic pressures, are the characteristic values that satisfy the characteristic matrix equation. The following equation that yields these roots is the compound sting divergence equation,

$$Q_{div} = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

where

$$\begin{aligned} A = & a_{21}a_{43}(C_{N\alpha}S)_1(C_{N\alpha}S)_2 + a_{21}a_{44}(C_{N\alpha}S)_1(C_{M\alpha}Sh)_2 \\ & + a_{22}a_{43}(C_{M\alpha}Sh)_1(C_{N\alpha}S)_2 + a_{22}a_{44}(C_{M\alpha}Sh)_1(C_{M\alpha}Sh)_2 \\ & - a_{23}a_{41}(C_{N\alpha}S)_1(C_{N\alpha}S)_2 - a_{23}a_{42}(C_{M\alpha}Sh)_1(C_{N\alpha}S)_2 \\ & - a_{24}a_{41}(C_{N\alpha}S)_1(C_{M\alpha}Sh)_2 - a_{24}a_{42}(C_{M\alpha}Sh)_1(C_{M\alpha}Sh)_2 \end{aligned}$$

$$B = -a_{21}(C_{N\alpha}S)_1 - a_{22}(C_{M\alpha}Sh)_1 - a_{43}(C_{N\alpha}S)_2 - a_{44}(C_{M\alpha}Sh)_2$$

$$C = +1$$

Although this equation yields two mathematical solutions only the minimum positive root is of concern and would be the critical divergence dynamic pressure for the sting system. A physical interpretation of this root is that it is the minimum dynamic pressure where the system becomes unstable and beyond which the aerodynamic loads on the models exceed the elastic restoring forces of the model support system.

CONCLUSION

The input data that is required for the compound sting divergence equation on the preceding page consists of a description of the rotational deflection influence coefficients and a definition of the aerodynamic loads. This data is needed before a sting divergence dynamic pressure can be evaluated using the compound sting divergence equation and it must be provided in the proper form. The influence coefficients have to be in terms of the rotational deflections per unit loads and the aerodynamic loads must be in terms of the rate of change of the normal force per unit angle of attack and the rate of change of the pitch moment per unit angle of attack. The reference center used to determine the respective aerodynamic loads must be relative to the same location that is used to evaluate the influence coefficients and the same units, either radians or degrees, need to be used throughout. The maximum slope of the normal force and pitch moment data can be used concurrently but an alternative approach would be to evaluate the worst sting divergence condition based on the effects of variations in the slope of the normal force and pitch moment data. The aerodynamic coefficient data can be digitized over selected ranges of angle of attack and a sting divergence dynamic pressure can be evaluated independently over each range. The worst case sting divergence condition would then be the minimum sting divergence dynamic pressure. This would provide less conservative results but should be acceptable provided the slopes for the aerodynamic coefficients change gradually over the selected ranges of angle of attack and provided the flexibility characteristics of the model support systems are similar. If there are any question about the relative differences in the slope or flexibility of the two models and model support systems, then the maximum slopes should be used.

The sample problem that is included in Appendix B illustrates the proper format for the input data, includes a step by step process of computing

results using the compound sting divergence equation, and includes a detailed discussion of the results. One of the steps included in the development of the input data is the process of transposing the aerodynamic loads from the aerodynamic reference center to the respective attachment centers of the models at the extreme ends of the model support system. A physical interpretation of the mathematical results is included together with a comparison to results obtained for a single sting configuration. The interpretation of the two sting divergence dynamic pressures that are obtained from the quadratic form of the compound sting divergence equation is that the minimum positive sting divergence dynamic pressure is the critical value that results when the aerodynamic loads on the two lifting bodies are additively combined and the maximum sting divergence dynamic pressure results from the aerodynamic loads on the two lifting bodies acting in opposing directions. Justification of this interpretation is through comparisons to results obtained from single sting configurations. A reduction in sting divergence dynamic pressure that would result from evaluating the sting divergence dynamic pressure from using the compound sting divergence equation as compared to the results from predicting sting divergence for a simple sting is 26.8 %. The reduction is substantial and indicates that for such parallel sting systems, the aeroelastic effects of compound stings need to be considered simultaneously.

APPENDIX A

LIST OF SYMBOLS

a_{11}	translation of 1 st body due to normal force on 1 st body
a_{12}	translation of 1 st body due to pitch moment on 1 st body
a_{13}	translation of 1 st body due to normal force on 2 nd body
a_{14}	translation of 1 st body due to pitch moment on 2 nd body
a_{21}	rotation of 1 st body due to normal force on 1 st body
a_{22}	rotation of 1 st body due to pitch moment on 1 st body
a_{23}	rotation of 1 st body due to normal force on 2 nd body
a_{24}	rotation of 1 st body due to pitch moment on 2 nd body
a_{31}	translation of 2 nd body due to normal force on 1 st body
a_{32}	translation of 2 nd body due to pitch moment on 1 st body
a_{33}	translation of 2 nd body due to normal force on 2 nd body
a_{34}	translation of 2 nd body due to pitch moment on 2 nd body
a_{41}	rotation of 2 nd body due to normal force on 1 st body
a_{42}	rotation of 2 nd body due to pitch moment on 1 st body
a_{43}	rotation of 2 nd body due to normal force on 2 nd body
a_{44}	rotation of 2 nd body due to pitch moment on 2 nd body
A	first general term in quadratic equation
B	second general term in quadratic equation
C	third general term in quadratic equation
$C_{N\alpha}$	change in normal force coefficient per unit angle of attack
$C_{M\alpha}$	change in pitch moment coefficient per unit angle of attack
h	reference mean aerodynamic chord of the respective body
I_i	sting sectional moment of inertia at x_i
M_1	pitch moment acting on 1 st body
M_2	pitch moment acting on 2 nd body
N_1	normal force acting on 1 st body
N_2	normal force acting on 2 nd body

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NF θ_i rotational deflection at x_i due to normal force
PM θ_i rotational deflection at x_i due to pitch moment
Q general term for dynamic pressure
 Q_{div} the critical divergence dynamic pressure
S reference planform area of the respective body
 x_i general sting coordinate along axis of sting (sting station)
 y_1 general translational deflection of 1st body
 y_2 general translational deflection of 2nd body
 θ_1 general rotational deflection of 1st body
 θ_2 general rotational deflection of 2nd body

APPENDIX B

SAMPLE PROBLEM

To illustrate results obtained from using the equations included in this report, a hypothetical sting system was developed using aerodynamic data and sting stiffness data from the Divergence Analysis Report for the Bodies of Revolution Model Support Systems, Reference 4. A representative parallel sting system was developed as shown in Figure 3 using the existing data for the E-1 and the E-3 bodies of revolution and respective stings. The deflection influence coefficients, that are required in the compound sting divergence equation in this report on page 7, were determined by applying finite element analysis techniques to a spreadsheet format. The spreadsheet data was validated by computing equivalent sting divergence pressures with the spreadsheet data and comparing the results to the results that were evaluated in the report. Duplicating the sting divergence results in the report with the spreadsheet data provided confidence in the data that was used to develop the compound sting system. Small differences in the results were encountered due to the results in the report including effects of aerodynamic drag, that were not included in the spreadsheet data, but were not considered to be significant.

The two stings that were selected from Reference 4 and coupled together to form the parallel sting system are identified in Figure 3. The larger E-3 sting was used in its entirety and the smaller E-1 sting was truncated at sting station 159.059 and hard coupled to the larger sting via a rigid coupling. The E-1 and E-3 bodies of revolution were positioned such that the nose of the smaller E-1 body was slightly aft of the nose of the larger E-3 body. These relative positions are representative of how two bodies would be mounted in a wind tunnel to study interference effects between adjacent models. The significance of identifying the relative positions of the bodies is in order to provide a reference center for determining the

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aerodynamic loads. Using the E-1 and E-3 bodies of revolution and sting systems allowed existing sting stiffness data to be used and provided good aerodynamic loadings that had already been evaluated. The sting data in the report had been optimized to provide sting divergence results that were accurate to within one half of one percent. The existing data provided a reliable source of data that represented an actual wind tunnel application.

The technique of developing an equivalent finite element model using a spreadsheet format was based on using successively evaluated rotations along cantilevered stings. Two independent sting systems were required: one for the larger unmodified sting system and one for the smaller truncated sting that was rigidly coupled to the aft end of the larger sting. Starting from the supported end, and repeating for each sting system, the rotations were independently evaluated for both a unit normal force and a unit pitch moment. The primary deflection influence coefficients for each of the two sting systems consisted of the end rotations of each of the respective sting systems due to the unit loads. The influence coefficients that represent the cross coupling terms were the same as the intermediate rotations of a particular sting system due to the unit loads that act on the other sting system. These intermediate rotations would be the same as the end rotations at the unloaded ends of the sting systems because the unloaded portion of the stings rotate as rigid bodies in the absence of end loads. The end loads produce elastic rotations from the point of application back to the supported end but for the unloaded portion of the sting forward of the rigid link that couples the two sting systems together, the rotations would not change. Since the aft ends of the two sting configurations were identical, the intermediate deflection influence coefficients for one sting system were actually determined by evaluating the elasticity of the other sting system. This was done to determine the cross coupling terms without having to develop

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new sting systems with the aerodynamic loads applied in the proper locations. Since the elasticity of the stings forward of the coupled joint does not effect the rotations aft of the coupled joint, separate math models of the stings to account for properly located loads were not considered necessary.

The spreadsheet data that was used to evaluate the deflection influence coefficients is listed at the end of this section of the report. The deflection influence coefficients that are used in the compound sting divergence equation can be found in the spreadsheet data. For the E-3 sting, the deflection coefficients are equivalent to the intermediate rotations at sting station 159.059 and to the end rotations at sting station 198.705. For the E-1/E-3 coupled sting the deflection coefficients are equivalent to the intermediate rotations at sting station 159.059 and to the end rotations at sting station 196.152. The deflection influence coefficients that were taken from the spreadsheet data are listed below with a brief description, where AOA is an abbreviated form of angle-of-attack:

$$a_{21} = 0.0025113005 \frac{\text{rad}}{\text{lb}} \quad (\text{AOA of E-1 due to unit normal force on E-1})$$

$$a_{22} = 0.0002140412 \frac{\text{rad}}{\text{in-lb}} \quad (\text{AOA of E-1 due to unit pitch moment on E-1})$$

$$a_{23} = 0.0000610159 \frac{\text{rad}}{\text{lb}} \quad (\text{AOA of E-1 due to unit normal force on E-3})$$

$$a_{24} = 0.0000012991 \frac{\text{rad}}{\text{in-lb}} \quad (\text{AOA of E-1 due to unit pitch moment on E-3})$$

$$a_{41} = 0.0000610159 \frac{\text{rad}}{\text{lb}} \quad (\text{AOA of E-3 due to unit normal force on E-1})$$

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$$a_{42} = 0.0000012991 \frac{\text{rad}}{\text{in-lb}} \quad (\text{AOA of E-3 due to unit pitch moment on E-1})$$

$$a_{43} = 0.0002085974 \frac{\text{rad}}{\text{lb}} \quad (\text{AOA of E-3 due to unit normal force on E-3})$$

$$a_{44} = 0.0000079136 \frac{\text{rad}}{\text{in-lb}} \quad (\text{AOA of E-3 due to unit pitch moment on E-3})$$

The aerodynamic loadings as used in the compound sting divergence equation are taken from Reference 4 and are given below. The transformation that is required to transpose the pitch moment aerodynamic load from the reference center of the model to the center of the model attachment to the sting is included to illustrate how to transfer the loads from the aerodynamic reference center to the center of the model attachment to the sting system, when the locations do not correspond.

$$\begin{aligned} (C_{N\alpha}S)_1 &= \left(\frac{\Delta C_{N1}}{\Delta \alpha} \right) S_{\text{ref1}} = \left(\frac{1.4324}{\text{rad}} \right) (3.37056 \text{ in}^2) \\ &= 4.828 \frac{\text{in}^2}{\text{rad}} \quad (\text{normal force loading for E-1 body}) \end{aligned}$$

$$\begin{aligned} (C_{M\alpha}Sh)_1 &= \left[\left(\frac{\Delta C_{M1}}{\Delta \alpha} \right) h_{\text{ref1}} - \left(\frac{\Delta C_{N1}}{\Delta \alpha} \right) \Delta h \right] S_{\text{ref1}} \\ &= \left[\left(\frac{1.2605}{\text{rad}} \right) (22.120 \text{ in}) - \left(\frac{1.4324}{\text{rad}} \right) (3.4633 \text{ in}) \right] (3.37056 \text{ in}^2) \\ &= 77.259 \frac{\text{in}^3}{\text{rad}} \quad (\text{pitch moment loading for E-1 body}) \end{aligned}$$

$$\begin{aligned} (C_{N\alpha}S)_2 &= \left(\frac{\Delta C_{N2}}{\Delta \alpha} \right) S_{\text{ref2}} = \left(\frac{1.4324}{\text{rad}} \right) (26.9703 \text{ in}^2) \\ &= 38.632 \frac{\text{in}^2}{\text{rad}} \quad (\text{normal force loading for E-3 body}) \end{aligned}$$

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$$\begin{aligned}
 (C_{M\alpha}Sh)_2 &= \left[\left(\frac{\Delta C_{M2}}{\Delta \alpha} \right) h_{ref2} - \left(\frac{\Delta C_{N2}}{\Delta \alpha} \right) \Delta h \right] S_{ref2} \\
 &= \left[\left(\frac{1.2605}{\text{rad}} \right) (62.565 \text{ in}) - \left(\frac{1.4324}{\text{rad}} \right) (12.2416 \text{ in}) \right] (26.9703 \text{ in}^2) \\
 &= 1654.043 \frac{\text{in}^3}{\text{rad}} \quad (\text{pitch moment loading for E-3 body})
 \end{aligned}$$

The following is an evaluation of the general terms in the quadratic form of the compound sting divergence equation. The first term is

$$\begin{aligned}
 A &= a_{21}a_{43}(C_{N\alpha}S)_1(C_{N\alpha}S)_2 + a_{21}a_{44}(C_{N\alpha}S)_1(C_{M\alpha}Sh)_2 \\
 &\quad + a_{22}a_{43}(C_{M\alpha}Sh)_1(C_{N\alpha}S)_2 + a_{22}a_{44}(C_{M\alpha}Sh)_1(C_{M\alpha}Sh)_2 \\
 &\quad - a_{23}a_{41}(C_{N\alpha}S)_1(C_{N\alpha}S)_2 - a_{23}a_{42}(C_{M\alpha}Sh)_1(C_{N\alpha}S)_2 \\
 &\quad - a_{24}a_{41}(C_{N\alpha}S)_1(C_{M\alpha}Sh)_2 - a_{24}a_{42}(C_{M\alpha}Sh)_1(C_{M\alpha}Sh)_2
 \end{aligned}$$

$$\begin{aligned}
 A &= [(0.0025113005)(0.0002085974)(4.828)(38.632) \\
 &\quad + (0.0025113005)(0.0000079136)(4.828)(1654.043) \\
 &\quad + (0.0002140412)(0.0002085974)(77.259)(38.632) \\
 &\quad + (0.0002140412)(0.0000079136)(77.259)(1654.043) \\
 &\quad - (0.0000610159)(0.0000610159)(4.828)(38.632) \\
 &\quad - (0.0000610159)(0.0000012991)(77.259)(38.632) \\
 &\quad - (0.0000012991)(0.0000610159)(4.828)(1654.043) \\
 &\quad - (0.0000012991)(0.0000012991)(38.632)(1654.043)] \frac{\text{in}^4}{\text{lb}^2}
 \end{aligned}$$

$$\begin{aligned}
 A &= [0.0000977062 + 0.0001587029 + 0.0001332608 + 0.0002164539 \\
 &\quad - 0.0000006944 - 0.0000002366 - 0.0000006330 - 0.0000002157] \frac{\text{in}^4}{\text{lb}^2}
 \end{aligned}$$

$$A = 0.0006043442 \frac{\text{in}^4}{\text{lb}^2}$$

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The second term is

$$B = -a_{21}(C_{N\alpha}S)_1 - a_{22}(C_{M\alpha}Sh)_1 - a_{43}(C_{N\alpha}S)_2 - a_{44}(C_{M\alpha}Sh)_2$$

$$B = [- (0.0025113005)(4.828) - (0.0002140412)(77.259) \\ - (0.0002085974)(38.632) - (0.0000079136)(1654.043)] \frac{\text{in}^2}{\text{lb}}$$

$$B = -0.0498090777 \frac{\text{in}^2}{\text{lb}}$$

and the third term is

$$C = +1$$

Substituting these values into the general compound sting divergence equation and solving for the two roots,

$$Q_{\text{div}} = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

$$Q_{\text{div}} = \frac{-(-.0498090777) \pm \sqrt{(-.0498090777)^2 - 4*(.00006043442)*(1)}}{2*(.00006043464)}$$

$$Q_{\text{div}} = \begin{cases} 34.61285 \frac{\text{lb}}{\text{in}^2} & (4984 \frac{\text{lb}}{\text{ft}^2}) & \text{minimum} \\ 47.80554 \frac{\text{lb}}{\text{in}^2} & (6884 \frac{\text{lb}}{\text{ft}^2}) & \text{maximum} \end{cases}$$

The minimum value of $4984 \frac{\text{lb}}{\text{ft}^2}$ is the critical divergence dynamic pressure in terms of "psf" for the compound sting illustrated in Figure 3. The other value of $6884 \frac{\text{lb}}{\text{ft}^2}$ is attributed to the aerodynamic loads on the two bodies being subtractively combined in lieu of being additively combined.

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The following can be used to illustrate the relative effect on dynamic pressure that results from combining the two single model support systems from Reference 4 into the parallel model support system that is shown in Figure 3. A comparative sting divergence dynamic pressure for the E-1 and the E-3 bodies can be independently evaluated from the spreadsheet data by combining the respective rotational deflection influence coefficients with the corresponding aerodynamic loads for each of the two bodies. This would be equivalent to independently testing each body separately on the parallel model support system without any aerodynamic effects from the sting segments. In the following computations Q_{div1} is for the sting system with the E-1 body only and Q_{div3} is for the sting system with the E-3 body only:

$$Q_{div1} = \frac{1}{a_{21}(C_{N\alpha}S)_1 + a_{22}(C_{M\alpha}Sh)_1}$$

$$Q_{div1} = \frac{1}{(.0025113005 \frac{\text{rad}}{\text{lb}})(4.828 \frac{\text{in}^2}{\text{rad}}) + (.0002140412 \frac{\text{rad}}{\text{in-lb}})(77.259 \frac{\text{in}^3}{\text{rad}})}$$

$$Q_{div1} = 34.890 \frac{\text{lb}}{\text{in}^2} \quad (5024 \frac{\text{lb}}{\text{ft}^2}) \quad \text{Divergence for E-1 body only}$$

$$Q_{div3} = \frac{1}{a_{43}(C_{N\alpha}S)_2 + a_{44}(C_{M\alpha}Sh)_2}$$

$$Q_{div3} = \frac{1}{(.0002085974 \frac{\text{rad}}{\text{lb}})(38.632 \frac{\text{in}^2}{\text{rad}}) + (.0000079136 \frac{\text{rad}}{\text{in-lb}})(1654.043 \frac{\text{in}^3}{\text{rad}})}$$

$$Q_{div3} = 47.286 \frac{\text{lb}}{\text{in}^2} \quad (6809 \frac{\text{lb}}{\text{ft}^2}) \quad \text{Divergence for E-3 body only}$$

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Note that the above 5024 psf sting divergence pressure for the E-1 body only and 6809 psf sting divergence pressure for the E-3 body only are nested between the sting divergence pressures of 4984 psf and 6884 psf that were obtained from the compound sting divergence equation. This supports the conclusions that the minimum sting divergence pressure results from additively combining the aerodynamic loads from the two bodies and that the maximum sting divergence pressure results from subtractively combining the aerodynamic loads. In addition it is presumed that the minimum sting divergence pressure obtained from the compound sting divergence equation is the result of adding the smaller E-1 body adjacent to the model support system for the larger E-3 body. Based on that presumption the minimum sting divergence dynamic pressure evaluated from the compound sting divergence equation represents a reduction in the sting divergence dynamic pressure of the E-3 sting system, the following expresses that reduction as a percentage:

$$\begin{aligned}\text{Percent Reduction in } Q_{\text{div}} &= \frac{Q_{\text{div}} \text{ (for compound sting)}}{Q_{\text{div3}} \text{ (for simple sting)}} \times 100\% \\ &= \frac{4984 \text{ psf}}{6809 \text{ psf}} \times 100 \% \\ &= 73.2 \%\end{aligned}$$

A comparison was not made to the E-1 sting system because the support for the E-1 body was considered more as addition to the E-3 sting system rather than as the basic sting configuration.

There are some small differences between the above sting divergence results and the results reported in Reference 4. The reasons for the differences are due to the model support systems not being modeled exactly the same and not including drag with the other aerodynamic loads. The above value of 5024 psf for the E-1 body only is lower than the corresponding sting

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divergence dynamic pressure of 5422 psf that was determined in Reference 4 because the sting system used in the spreadsheet data is more flexible. The forward end of the E-1 sting was extended, to suit the parallel sting configuration modeled with the spreadsheet data, and the affects of the additional flexibility overpowered the effects of excluding aerodynamic drag, resulting in a lower sting divergence dynamic pressure. For the E-3 body the above sting divergence dynamic pressure of 6809 psf is higher than value of 6473 psf reported in Reference 4. The E-3 sting is basically the same in both configurations and the higher value obtained from the spreadsheet data is attributed to not having included the effects of aerodynamic drag. The effects of drag were excluded from the spreadsheet data in order to simplify the sample problem and were not considered necessary to illustrate the new technique included in this report. The only difference in the sting configurations was the method of supporting the stings. For the spreadsheet data the sting was cantilevered and in Reference 4 the sting was supported by a more comprehensive definition of the aft end of the model support system. The effect of the differences in the sting configurations was insignificant in comparison to the effects of drag and would only slightly lower the divergence pressure predicted by the spreadsheet data.

The following pages are the referenced listing of the LOTUS spreadsheet data that was used to evaluate the deflection influence coefficients.

Rotational deflections for NTF-107 balance, fwd end E-1 sting, aft end E-3 sting, and NTF stub sting

x_i	I_i	NF θ_i	PM θ_i	x_i	I_i	NF θ_i	PM θ_i	x_i	I_i	NF θ_i	PM θ_i
63.25	2691.925	0.000000063	0.000000000	127.066	3.92026	0.0000028453	0.0000000371	176.984	0.071773	0.0002275268	0.0000067706
66.356	2308.086	0.000000000	4.74197E-14	129.668	3.92026	0.0000046271	0.0000000625	177.673	0.054123	0.0002364571	0.0000071884
69.463	1966.824	0.0000000136	1.02901E-10	131.27	3.92026	0.0000063431	0.0000000878	178.363	0.038796	0.0002480586	0.0000077492
75.569	1664.868	0.0000000298	2.31245E-10	134.872	3.92026	0.0000079932	0.0000001131	179.052	0.028346	0.0002634933	0.000008551
75.676	1399.077	0.0000000301	0.0000000002	137.474	3.92026	0.0000095773	0.0000001385	179.355	0.023223	0.0002712411	0.0000089183
78.782	1166.441	0.0000000414	0.0000000003	140.076	3.92026	0.0000110955	0.0000001638	179.658	0.022908	0.0002800862	0.0000093791
81.888	964.081	0.0000000545	0.0000000004	142.678	3.92026	0.0000125478	0.0000001891	179.961	0.019832	0.0002902403	0.0000099164
84.995	789.248	0.0000000701	0.0000000006	143.54	3.46703	0.0000130431	0.0000001981	180.263	0.018835	0.0003015296	0.0000105451
88.101	639.326	0.0000000887	0.0000000007	144.402	3.05424	0.0000135953	0.0000002081	180.566	0.018188	0.0003135659	0.0000112907
91.207	511.827	0.0000001112	0.0000000009	145.264	2.67943	0.0000142136	0.0000002196	180.869	0.018182	0.0003311562	0.0000121785
94.314	404.396	0.0000001386	0.0000000012	146.127	2.34019	0.0000149093	0.0000002327	181.172	0.009831	0.0003503914	0.0000132447
97.42	314.808	0.0000001725	0.0000000015	146.989	2.03418	0.0000156937	0.0000002478	181.475	0.008067	0.0003728527	0.000014537
100.527	240.968	0.0000002151	0.000000002	147.851	1.75918	0.0000165933	0.0000002651	181.778	0.006547	0.0003998799	0.0000161195
103.633	180.912	0.0000002694	0.0000000025	148.713	1.513	0.0000175933	0.0000002852	182.081	0.005248	0.0004272727	0.0000180803
104.008	117.626	0.0000002785	0.0000000026	149.575	1.29356	0.0000187593	0.0000003087	182.383	0.004146	0.0004734957	0.0000205342
104.508	84.5719	0.0000002963	0.0000000028	150.437	1.09886	0.0000200987	0.0000003362	182.686	0.003422	0.0005239556	0.0000236732
105.25	74.116	0.0000003298	0.0000000032	151.3	0.926947	0.0000216543	0.0000003687	182.989	0.003452	0.0005886364	0.0000277495
105.35	180.912	0.0000003298	0.0000000032	152.162	0.775984	0.0000234694	0.0000004074	183.292	0.003452	0.0006728824	0.0000331622
115.25	180.912	0.0000005164	0.0000000053	153.886	0.64419	0.0000256059	0.0000004537	184.337	0.003452	0.0007990051	0.0000350636
115.35	139.442	0.0000005164	0.0000000053	154.748	0.529872	0.000028142	0.0000005097	185.382	0.003452	0.0009990051	0.0000350636
116.275	146.481	0.0000005391	0.0000000055	155.61	0.43141	0.0000311805	0.0000005782	186.427	0.003452	0.0013022408	0.0000369654
116.934	161.27	0.0000005525	0.0000000057	156.473	0.347266	0.0000348886	0.0000006627	187.472	0.003452	0.0018400513	0.0000407678
117.175	181.619	0.0000005569	0.0000000058	157.335	0.275979	0.0000393882	0.0000007684	188.517	0.003452	0.0020616408	0.0000414568
117.899	162.902	0.0000005691	0.0000000059	158.197	0.216169	0.0000449571	0.0000009021	188.624	0.003452	0.0020788084	0.0000431508
118.279	145.67	0.000000576	0.0000000061	158.059	0.166531	0.0000519362	0.000001074	188.731	0.003452	0.0020900571	0.0000442725
118.67	129.842	0.0000005828	0.0000000062	159.059	0.125941	0.0000610159	0.0000012991	188.838	0.003452	0.0020964555	0.0000449175
119.451	115.34	0.0000006021	0.0000000063	159.748	0.089	0.0000680341	0.0000015031	188.945	0.003452	0.0020997467	0.0000452529
119.841	102.087	0.0000006129	0.0000000065	160.438	0.06323	0.000076923	0.0000017074	189.052	0.003452	0.0021021959	0.0000455052
120.232	90.092	0.0000006251	0.0000000066	161.127	0.0438	0.0000846598	0.0000019114	189.332	0.003452	0.0021086027	0.0000461787
120.622	79.0356	0.0000006389	0.0000000068	161.817	0.0289	0.000092267	0.0000021158	189.428	0.003452	0.0021121333	0.0000465573
121.012	69.0964	0.0000006546	0.000000007	162.506	0.0189	0.0000997324	0.0000023198	191.67	0.003452	0.0022956541	0.00004690586
121.403	60.1244	0.0000006725	0.0000000072	163.195	0.01289	0.0001070373	0.0000025238	193.912	0.003452	0.0024287269	0.00004915599
121.793	52.0549	0.0000006929	0.0000000075	163.885	0.0089	0.000114222	0.0000027281	195.032	0.003452	0.0024763085	0.00005028006
121.84	44.8252	0.0000007166	0.0000000078	164.574	0.0063	0.000121256	0.0000029321	196.152	0.003452	0.0025113005	0.00005140412
122.574	38.3748	0.0000007439	0.0000000082	165.264	0.0043	0.0001281585	0.0000031364				
122.965	32.6457	0.0000007758	0.0000000086	165.953	0.00289	0.0001349107	0.0000033404				
123.206	29.2654	0.0000007983	0.0000000089	166.643	0.00189	0.0001415118	0.0000035448				
123.355	27.5516	0.0000008133	0.0000000091	167.332	0.001289	0.0001480027	0.0000037488				
123.746	23.2055	0.0000008574	0.0000000097	168.021	0.00089	0.0001543311	0.0000039528				
124.116	19.1138	0.0000009098	0.0000000104	168.711	0.00059	0.0001605317	0.0000041571				
124.526	15.3334	0.0000009726	0.0000000112	170.09	0.00039	0.000165807	0.0000043611				
124.917	13.0173	0.0000010489	0.0000000122	170.779	0.000289	0.0001724976	0.0000045654				
125.307	10.5211	0.0000011442	0.0000000135	171.468	0.000189	0.0001782652	0.0000047694				
125.698	8.40221	0.0000012574	0.0000000151	172.158	0.0001289	0.0001838923	0.0000049735				
126.088	6.63064	0.0000014017	0.0000000171	172.847	0.000089	0.0001893867	0.0000051778				
126.479	5.13852	0.0000015855	0.0000000196	173.537	0.000059	0.0001947324	0.0000053816				
126.869	3.92026	0.0000018323	0.0000000229	174.226	0.000039	0.0002050094	0.0000055861				
127.066	0.52728	0.0000018223	0.0000000229	175.009	0.000025	0.0002117367	0.0000057901				
				175.605	0.000018	0.0002148428	0.0000062036				
				176.294	0.000012	0.0002204827	0.0000064514				

Rotational Deflections for NTF-105 Balance, E-3 Sting, and NTF Stub Sting

x_i	I_i	NF θ_i	PM θ_i	x_i	I_i	NF θ_i	PM θ_i	x_i	I_i	NF θ_i	PM θ_i
63.25	2691.925	0.000000063	0.000000000	127.066	3.92026	0.0000028453	0.0000000371	182.624	1.02643	0.0001930103	0.0000053785
66.356	2308.086	0.0000000136	4.74197E-11	129.668	3.92026	0.00000046271	0.0000000625	184.074	1.02643	0.0001930103	0.0000053785
69.464	1966.824	0.00000000136	1.02901E-10	132.27	3.92026	0.00000005331	0.0000000878	185.524	1.02643	0.0001945881	0.0000054864
75.569	1664.868	0.0000000298	2.31245E-10	134.872	3.92026	0.00000079932	0.0000001131	185.524	0.881196	0.0001945881	0.0000054864
75.576	1399.077	0.0000000301	0.0000000002	137.474	3.92026	0.0000005773	0.0000001385	187.705	0.782039	0.0001957983	0.0000055865
78.782	1166.441	0.0000000414	0.0000000003	140.076	3.92026	0.0000110955	0.0000001638	187.705	0.180416	0.0001957983	0.0000055865
81.888	964.081	0.0000000545	0.0000000004	142.678	3.92026	0.00001125478	0.0000001891	190.423	0.180416	0.0002013419	0.0000061615
84.995	789.248	0.0000000701	0.0000000006	143.54	3.92026	0.0000130431	0.0000001981	193.14	0.180416	0.0002053215	0.0000067363
88.101	639.326	0.0000000887	0.0000000007	144.402	3.92026	0.0000135953	0.0000002081	195.923	0.180416	0.000207787	0.000007325
91.207	511.827	0.0000001112	0.0000000009	145.264	3.92026	0.0000142136	0.0000002196	198.705	0.180416	0.0002085974	0.0000079136
94.314	404.396	0.0000001386	0.0000000012	146.127	2.34019	0.0000149093	0.0000002327				
97.42	314.808	0.0000001725	0.0000000015	146.989	2.03418	0.0000156937	0.0000002478				
100.527	240.968	0.0000002151	0.000000002	147.851	1.75918	0.0000158833	0.0000002651				
103.633	180.912	0.0000002694	0.0000000025	148.713	1.513	0.0000175973	0.0000002852				
104.008	117.626	0.0000002785	0.0000000026	149.575	1.29356	0.0000187593	0.0000003087				
104.508	84.6719	0.0000003293	0.0000000028	150.437	1.09886	0.0000200987	0.0000003362				
105.25	74.116	0.0000003298	0.0000000032	151.3	0.926947	0.0000216543	0.0000003687				
105.25	180.912	0.0000003298	0.0000000032	152.162	0.775984	0.0000234694	0.0000004074				
115.25	139.442	0.0000005164	0.0000000053	153.024	0.64419	0.0000256059	0.0000004537				
116.275	146.481	0.0000005391	0.0000000055	153.886	0.529872	0.000028142	0.0000005097				
116.234	161.27	0.0000005525	0.0000000057	154.748	0.43141	0.0000311805	0.0000005782				
117.175	181.619	0.0000005569	0.0000000058	155.61	0.347266	0.0000348586	0.0000006627				
117.889	181.619	0.0000005691	0.0000000059	156.473	0.275979	0.0000393682	0.0000007684				
118.279	162.902	0.000000576	0.000000006	157.335	0.216169	0.0000449571	0.0000009021				
118.67	145.67	0.0000005838	0.0000000061	158.197	0.166531	0.0000519962	0.000001074				
119.06	129.842	0.0000005924	0.0000000062	159.059	0.125841	0.0000610159	0.0000012991				
119.451	115.34	0.0000006021	0.0000000063	160.146	0.125841	0.0000739076	0.0000016288				
119.841	102.087	0.0000006129	0.0000000065	161.232	0.125841	0.0000864296	0.0000019582				
120.232	90.0092	0.0000006251	0.0000000066	162.319	0.125841	0.0000986049	0.0000022879				
120.622	79.0356	0.0000006389	0.0000000068	163.405	0.125841	0.0001104111	0.0000026172				
121.012	69.0964	0.0000006546	0.000000007	164.492	0.125841	0.00012187	0.0000029469				
121.403	60.1244	0.0000006725	0.0000000072	165.578	0.125841	0.0001329605	0.0000032763				
121.793	52.0549	0.0000006929	0.0000000075	166.665	0.125841	0.0001437029	0.000003606				
122.184	44.8252	0.0000007166	0.0000000078	167.751	0.125841	0.0001540776	0.0000039354				
122.574	38.3748	0.0000007439	0.0000000082	168.838	0.125841	0.0001641037	0.0000042651				
122.965	32.6457	0.0000007758	0.0000000086	169.924	0.125841	0.0001737626	0.0000045945				
123.356	29.516	0.0000007983	0.0000000089	170.288	0.125841	0.0001837492	0.0000049523				
123.746	27.6575	0.0000008133	0.0000000091	170.651	0.202562	0.0001837492	0.0000049523				
124.136	23.2055	0.0000008574	0.0000000097	171.015	0.251762	0.0001837492	0.0000049523				
124.526	19.3138	0.0000009098	0.0000000104	171.379	0.309303	0.0001837492	0.0000049523				
124.917	13.0173	0.0000010489	0.0000000112	171.742	0.376079	0.0001837492	0.0000049523				
125.307	10.5211	0.000001142	0.0000000135	172.106	0.453028	0.0001837492	0.0000049523				
125.698	8.40221	0.0000012574	0.0000000151	172.469	0.541136	0.0001837492	0.0000049523				
126.084	6.62064	0.0000014017	0.0000000171	172.833	0.641432	0.0001837492	0.0000049523				
126.479	5.13852	0.0000015855	0.0000000196	173.197	0.754991	0.0001837492	0.0000049523				
126.869	3.92026	0.0000018223	0.0000000229	173.56	0.882935	0.0001837492	0.0000049523				
127.066	0.52728	0.00000238453	0.0000000371	173.924	1.02643	0.0001837492	0.0000049523				
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				194.035	1.02643						

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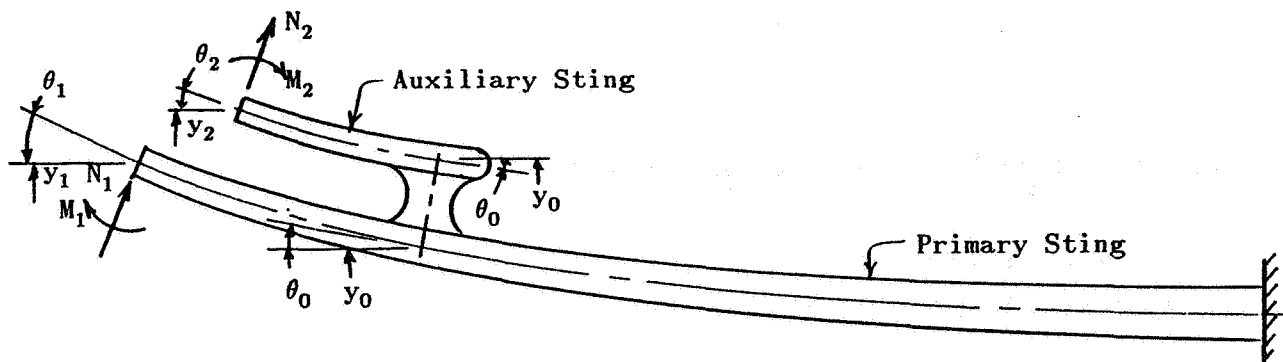


Figure 1. Parallel Sting System

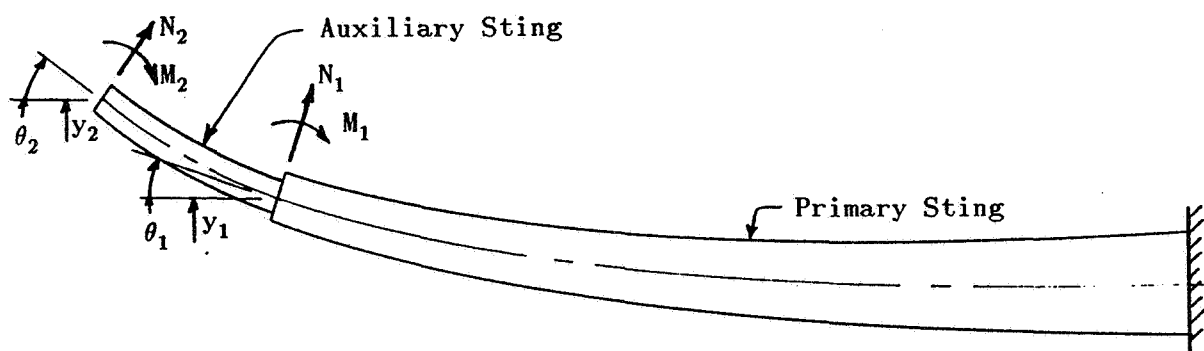


Figure 2. Tandem Sting System

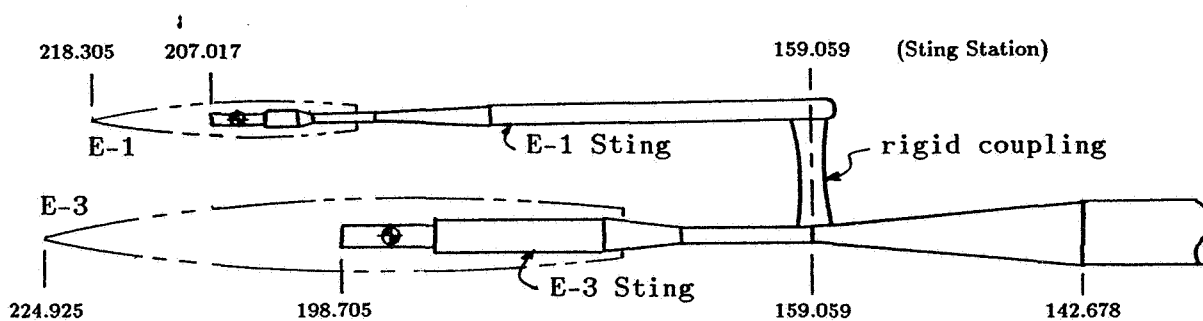


Figure 3. Parallel Mounting of Two Bodies of Revolution

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13. ABSTRACT (Maximum 200 words) An analytical technique is presented herein that can be used to evaluate the critical sting divergence dynamic pressure of compound sting system. A compound sting configuration can be described as two model support systems that are elastically coupled together such that the loads on one of the models affects the deflections of the other. The technique of evaluating divergence of compound stings is applicable to configurations that can be identified as either a tandem or a parallel sting system, depending on the relative position of the models. An example of a tandem sting system is a model with a nose that is decoupled and independently mounted to the main fuselage body in order to measure loads on a forward strake or on a canard. An example of a parallel sting system is a model support system that supports adjacent models for studying flow interference effects between the two models. A closed form solution for evaluating the divergence dynamic pressure of such compound stings is presented in the report. The input that is required include the rotational deflection influence coefficients, that can be determined either empirically or analytically, and the aerodynamic characteristics, that define the rate of change in the aerodynamic loads in terms of per unit angle-of-attack for the respective models.				
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